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## Generalized supersymmetry and sigma models



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### ABSTRACT

In this paper, we discuss the generalizations of exact supersymmetries present in the supersymmetrized sigma models. These generalizations are made by making the supersymmetric transformation parameter field-dependent. Remarkably, the supersymmetric effective actions emerge naturally through the Jacobian associated with the generalized supersymmetry transformations. We explicitly demonstrate these for two different supersymmetric sigma models, namely, one-dimensional sigma model and topological sigma model for hyperinstantons on quaternionic manifold.

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### 1. Introduction

Supersymmetry is one of the most important concepts in modern theoretical physics, especially in the search of unified theories beyond the standard model [1]. In particle physics, for example, the supersymmetric standard model predicts the existence of a superpartner for every particle in the standard model. However, theoretical understanding of supersymmetry is quite far from complete. To examine the non-perturbative aspects of supersymmetric standard model, the utilization of the so-called space–time lattice simulation method is quite obscure as the theory involves many different scales. Supersymmetry is also relevant in string theories though it is quite far from the real experimental world. The advantage of superstring theories (those string models which also incorporate supersymmetry) is that it does not predict the existence of a bad behaving particle called the Tachyon. In particle theory, supersymmetry finds a way to stabilize the hierarchy between the unification scale and the electroweak scale or the Higgs boson mass. Supersymmetry models are also considered as a natural dark matter candidate [2].

Since it encompasses both theoretical and phenomenological interests, some serious attempts have been made to study supersymmetric theories [3,4]. But these attempts encountered some problems like supersymmetry breaking or fine-tuning. Recent developments have been made in the construction of lattice actions which possess a subset of the supersymmetries of the continuum theory and have a Poincaré invariant continuum limit [5]. The presence of the exact supersymmetry provides a way to ob-

tain the continuum limit with no fine tuning or fine tuning much smaller than conventional lattice constructions. The remarkable feature of the presence of exact supersymmetry is that it reduces and in some cases eliminates the need for fine tuning to achieve a continuum limit invariant under the full supersymmetry of the target theory [5–7]. However, the construction of the supersymmetric non-linear sigma model with  $O(N)$  target manifold was first made by Witten [8] and then by P. Di Vecchia and S. Ferrara [9] who describe the spontaneous breaking of chiral symmetry and the dynamical generation of particle masses [10–13]. Subsequently, the geometric interpretation of supersymmetric sigma models was classified in terms of BRST operator [14,15]. These sigma models are described by maps between a two-dimensional space called the world-sheet and some target space, taken to be a manifold in this setting. The connections of supersymmetry and geometry became more stronger after Witten's seminal construction of the so-called topological twist [16]. The motivation behind the twist is that in a topological field theory one can compute certain physical quantities more easily than in the original theory, where we sometimes lack the tools to compute them exactly. The topological sigma models in four dimensions are also used in the study of triholomorphic maps on hyper-Kähler manifolds [17]. A naive discussion of gauge invariant topological field theory is presented in BRST-BV framework [18].

On the other hand, generalization of BRST transformation by making the infinitesimal parameter finite and field-dependent was first developed in [19] and is known as finite field-dependent BRST (FFBRST) transformation. Such generalizations have found various applications in gauge field theories as well as in M-theory [19–30]. However, this generalization of BRST technique has, as yet, not been done for supersymmetry. Considering the deep connection between BRST and supersymmetry, we feel that this is a glaring omission.

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The aim of the present paper is to investigate the features of generalized supersymmetry in the framework of FFBRST formulation. Specifically, we consider supersymmetric sigma model and supersymmetric topological sigma model in a gauge invariant framework. Further, we discuss the generalizations of supersymmetries present in the theory in a detailed way. These generalizations are made by making the infinitesimal transformation parameter finite and field-dependent. Further, we stress the significant features of this generalized supersymmetry. For instance, we find that while the effective actions are invariant under generalized supersymmetry, the measures of path integrals are not. The obvious reason for this is that the path integral measure changes non-trivially. This non-trivial Jacobian plays a significant role in the formation of supersymmetric actions for sigma models. We show that the path integral measure under generalized supersymmetry transformation with some specific choices of parameter reproduces exactly the same effective actions as the original theories. In other words, the supersymmetric actions proposed in the literature [6,17] may be systematically obtained within the framework of FFBRST transformations. We analyse results in one-dimensional supersymmetric sigma model and in supersymmetric topological sigma model where the gauge-fixing is provided by the triholomorphic instanton condition. Even though we establish the results with the help of specific examples but this works for a general supersymmetric invariant theory.

The paper is organized in four sections. First, we provide the mechanism to generalize the supersymmetry in FFBRST framework in Section 2. In Section 3, which is the main section of the paper, we show that the Jacobians of the functional measures for FFBRST transformations with judicious choices of the transformation parameters naturally yield the supersymmetric actions for sigma models. We draw concluding remarks in the last section.

## 2. Generalized supersymmetric BRST transformation

In this section, we briefly review the generalized supersymmetric BRST formulation of pure gauge theories by making the infinitesimal parameter finite and field-dependent. It is a supersymmetric generalization of finite field-dependent BRST (FFBRST) transformation originally advocated in [19] for the non-supersymmetric cases. We first present the general methodology for the standard Maxwell theory in Euclidean space-time. For this purpose, let us start by defining the partition function for BRST invariant Maxwell theory in four dimensions as following

$$Z_M = \int \mathcal{D}A_\mu \mathcal{D}c \mathcal{D}\bar{c} \mathcal{D}B e^{-S_M}, \quad (1)$$

where the effective action  $S_M$  in Lorentz gauge is defined by

$$S_M = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} B^2 - B \partial_\mu A^\mu + \partial_\mu \bar{c} \partial^\mu c \right]. \quad (2)$$

Here  $B$ ,  $c$  and  $\bar{c}$  are Nakanishi–Lautrup, ghost and anti-ghost fields, respectively. This effective action as well as the partition function are invariant under usual BRST transformations

$$\begin{aligned} \delta_b A_\mu(x) &= \partial_\mu c(x) \delta\Lambda, \\ \delta_b c(x) &= 0, \\ \delta_b \bar{c}(x) &= B(x) \delta\Lambda, \\ \delta_b B(x) &= 0, \end{aligned} \quad (3)$$

where  $\delta\Lambda$  is an infinitesimal, anticommuting and global parameter. Most of the features of the BRST transformation do not depend on whether the parameter  $\delta\Lambda$  is (i) finite or infinitesimal, (ii) field-dependent or not, as long as it is anticommuting and space-time

independent. These observations give us a freedom to generalize the BRST transformation by making the parameter,  $\delta\Lambda$ , finite and field-dependent without affecting its properties. To generalize such transformation we start by making the infinitesimal parameter field-dependent with introduction of an arbitrary parameter  $\kappa$  ( $0 \leq \kappa \leq 1$ ). We allow the generic fields,  $\Phi(x, \kappa)$ , to depend on  $\kappa$  in such a way that  $\Phi(x, \kappa = 0) = \Phi(x)$  and  $\Phi(x, \kappa = 1) = \Phi'(x)$ , the transformed field.

The usual infinitesimal transformation, thus can be written generically as [19]

$$\begin{aligned} \frac{dA_\mu(x, \kappa)}{d\kappa} &= \partial_\mu c(x) \Theta'[\Phi(x, \kappa)], \\ \frac{dc(x, \kappa)}{d\kappa} &= 0, \\ \frac{d\bar{c}(x, \kappa)}{d\kappa} &= B(x) \Theta'[\Phi(x, \kappa)], \\ \frac{dB(x, \kappa)}{d\kappa} &= 0, \end{aligned} \quad (4)$$

where the  $\Theta'[\Phi(x, \kappa)]$  is the infinitesimal but field-dependent parameter. The FFBRST transformation ( $\delta_f$ ) then can be constructed by integrating such infinitesimal transformation from  $\kappa = 0$  to  $\kappa = 1$ , as

$$\begin{aligned} \delta_f A_\mu(x) &= A_\mu(x, \kappa = 1) - A_\mu(x, \kappa = 0) = \partial_\mu c(x) \Theta[\Phi(x)], \\ \delta_f c(x) &= c(x, \kappa = 1) - c(x, \kappa = 0) = 0, \\ \delta_f \bar{c}(x) &= \bar{c}(x, \kappa = 1) - \bar{c}(x, \kappa = 0) = B(x) \Theta[\Phi(x)], \\ \delta_f B(x) &= B(x, \kappa = 1) - B(x, \kappa = 0) = 0, \end{aligned} \quad (5)$$

where [19]

$$\Theta[\Phi(x)] = \int_0^1 d\kappa' \Theta'[\Phi(x, \kappa')] \quad (6)$$

is the finite field-dependent parameter. Such a generalized transformation with finite field-dependent parameter is a symmetry of the effective action  $S_M$ , i.e.,

$$\delta_f S_M = (s_b S_M) \Theta = 0, \quad (7)$$

where  $s_b$  is Slavnov variation. Let us explicitly show the invariance of the Maxwell term. Under the transformations (5), the Maxwell pieces changes as,

$$\begin{aligned} \delta_f (F_{\mu\nu} F^{\mu\nu}) &= 4F_{\mu\nu} \delta_f \partial^\mu A^\nu, \\ &= 4F_{\mu\nu} \partial^\mu [\partial^\nu c \Theta], \\ &= 0. \end{aligned} \quad (8)$$

Since the FFBRST parameter  $\Theta$  is space-time independent, the derivative acts only on the variable  $c$ . By symmetry this term vanishes. Hence the Maxwell piece remains invariant. Although the action remains invariant, the functional measure is not invariant under such a transformation as the Grassmann parameter is field-dependent in nature. The Jacobian,  $J(\kappa)$ , of path integral measure changes non-trivially and can be replaced as [19]

$$J(\kappa) \mapsto e^{-S_1[\Phi(x, \kappa)]}, \quad (9)$$

if and only if the following condition is satisfied as we do not want any numerical change in the path integral measure [19]

$$\int \mathcal{D}\Phi(x) \left[ \frac{d}{d\kappa} \ln J(\kappa) + \frac{dS_1[\Phi(x, \kappa)]}{d\kappa} \right] e^{-S_1[\Phi(x, \kappa)]} = 0, \quad (10)$$

where  $S_1[\Phi]$  is some local functional of fields satisfying an initial boundary condition

$$S_1[\Phi]_{\kappa=0} = 0. \quad (11)$$

Furthermore, the infinitesimal change of the logarithm of  $J(\kappa)$  can be calculated from the formula [19]:

$$\begin{aligned} \frac{d}{d\kappa} \ln J(\kappa) &= - \int d^4x \left[ \partial_\mu c(x) \frac{\partial \Theta'[\Phi(x, \kappa)]}{\partial A_\mu(x, \kappa)} - B(x) \frac{\partial \Theta'[\Phi(x, \kappa)]}{\partial \bar{c}(x, \kappa)} \right]. \end{aligned} \quad (12)$$

For a particular choice of  $\Theta'[\Phi(x, \kappa)]$ , given by

$$\Theta'[\Phi(x, \kappa)] = - \int d^4x \bar{c} [\partial_\mu A^\mu(x, \kappa) - \eta_\mu A^\mu(x, \kappa)], \quad (13)$$

the expression in (12) reduces to

$$\begin{aligned} \frac{d}{d\kappa} \ln J(\kappa) &= \int d^4x [-\partial_\mu c \partial^\mu \bar{c} - \partial_\mu c \eta^\mu \bar{c} - B \partial_\mu A^\mu + B \eta_\mu A^\mu], \\ &= \int d^4x [\partial_\mu \bar{c} \partial^\mu c + \eta^\mu \bar{c} \partial_\mu c - B \partial_\mu A^\mu + B \eta_\mu A^\mu]. \end{aligned} \quad (14)$$

Now, an ansatz for the functional  $S_1[\Phi]$  is taken as

$$\begin{aligned} S_1 &= \int d^4x [\zeta_1(\kappa) B \partial_\mu A^\mu + \zeta_2(\kappa) B \eta_\mu A^\mu + \zeta_3(\kappa) \partial_\mu \bar{c} \partial^\mu c \\ &\quad + \zeta_4(\kappa) \eta^\mu \bar{c} \partial_\mu c], \end{aligned} \quad (15)$$

where  $\zeta_i$  ( $i = 1, 2, 3, 4$ ) are arbitrary constant parameters constrained by

$$\zeta_i(\kappa = 0) = 0, \quad (16)$$

so that the requirement (11) holds.

To satisfy the essential condition (10), we calculate the  $dS_1/d\kappa$  by employing (4) as follows:

$$\begin{aligned} \frac{dS_1}{d\kappa} &= \int d^4x \left[ \frac{d\zeta_1}{d\kappa} B \partial_\mu A^\mu + \frac{d\zeta_2}{d\kappa} B \eta_\mu A^\mu + \frac{d\zeta_3}{d\kappa} \partial_\mu \bar{c} \partial^\mu c \right. \\ &\quad \left. + \frac{d\zeta_4}{d\kappa} \eta^\mu \bar{c} \partial_\mu c + (\zeta_1 + \zeta_3) B (\partial_\mu \partial^\mu c) \Theta' \right. \\ &\quad \left. + (\zeta_2 - \zeta_4) B (\eta_\mu \partial^\mu c) \Theta' \right]. \end{aligned} \quad (17)$$

The condition (10) along with Eqs. (14) and (17) leads to

$$\begin{aligned} \int d^4x &\left[ \left( \frac{d\zeta_1}{d\kappa} - 1 \right) B \partial_\mu A^\mu + \left( \frac{d\zeta_2}{d\kappa} + 1 \right) B \eta_\mu A^\mu \right. \\ &\quad \left. + \left( \frac{d\zeta_3}{d\kappa} + 1 \right) \partial_\mu \bar{c} \partial^\mu c + \left( \frac{d\zeta_4}{d\kappa} + 1 \right) \eta^\mu \bar{c} \partial_\mu c \right. \\ &\quad \left. + (\zeta_1 + \zeta_3) B (\partial_\mu \partial^\mu c) \Theta' + (\zeta_2 - \zeta_4) B (\eta_\mu \partial^\mu c) \Theta' \right] = 0. \end{aligned} \quad (18)$$

The last two non-local ( $\Theta'$ -dependent) terms disappear from the above equation for  $(\zeta_1 + \zeta_3) = (\zeta_2 - \zeta_4) = 0$ . However, the disappearance of local terms yields the following differential equations

$$\begin{aligned} \frac{d\zeta_1}{d\kappa} - 1 &= 0, & \frac{d\zeta_2}{d\kappa} + 1 &= 0, \\ \frac{d\zeta_3}{d\kappa} + 1 &= 0, & \frac{d\zeta_4}{d\kappa} + 1 &= 0. \end{aligned} \quad (19)$$

The solutions of the above equations satisfying the boundary conditions (16) are

$$\zeta_1 = \kappa, \quad \zeta_2 = -\kappa, \quad \zeta_3 = -\kappa, \quad \zeta_4 = -\kappa. \quad (20)$$

With these identifications, the functional  $S_1[\Phi(x, \kappa), \kappa]$  has the form

$$\begin{aligned} S_1[\Phi(x, \kappa), \kappa] &= \int d^4x [\kappa B \partial_\mu A^\mu - \kappa B \eta_\mu A^\mu - \kappa \partial_\mu \bar{c} \partial^\mu c - \kappa \eta^\mu \bar{c} \partial_\mu c], \end{aligned} \quad (21)$$

which vanishes at  $\kappa = 0$ . Now, by adding this  $S_1[\Phi(x, \kappa), \kappa]$  to  $S_M$  given in (2), we obtain

$$\begin{aligned} S_M + S_1[\Phi(x, \kappa), \kappa] &= \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} B^2 - (1 - \kappa) B \partial_\mu A^\mu \right. \\ &\quad \left. + (1 - \kappa) \partial_\mu \bar{c} \partial^\mu c - \kappa B \eta_\mu A^\mu - \kappa \eta^\mu \bar{c} \partial_\mu c \right]. \end{aligned} \quad (22)$$

At  $\kappa = 0$ , the above expression reduces to

$$\begin{aligned} S_M + S_1[\Phi(x, 0), 0] &= \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} B^2 - B \partial_\mu A^\mu + \partial_\mu \bar{c} \partial^\mu c \right], \end{aligned} \quad (23)$$

which is the original theory in Lorentz gauge. However, at  $\kappa = 1$  (under FFBRST transformation) the expression (22) within a functional integration effectively reduces to the Maxwell action in axial gauge as given below

$$\begin{aligned} S_M + S_1[\Phi(x, 1), 1] &= \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} B^2 - B \eta_\mu A^\mu - \eta^\mu \bar{c} \partial_\mu c \right]. \end{aligned} \quad (24)$$

This shows that the FFBRST formulation is able to connect two different gauge fixed versions of the Maxwell theory. Incidentally, this was the original motivation for developing the FFBRST transformation.

A natural question that arises in this context is the possibility of generating the action itself through FFBRST formulation. To answer this question it is useful to ponder on the structure of the Jacobian (13). This involves terms that are subsequently interpreted as a combination of gauge fixing and ghost terms. Such combinations, which are BRST exact, appropriately modify the structure to connect the Maxwell theory in distinct gauges. It is clear, therefore, that since the Jacobian is BRST exact, this by itself would fail to generate the Maxwell action simply because it is not BRST exact. Hence, in order for the Jacobian to reproduce the whole action, that particular action must be BRST exact. Such a possibility occurs for the supersymmetric sigma models. To implement these notions, therefore, it is essential to first extend the FFBRST formulation to include supersymmetry.

To generalize the FFBRST formulation for supersymmetric transformation, let us write the usual supersymmetric transformation for a collective field  $\Phi$  of sigma models,

$$\delta\Phi = \mathcal{R}[\Phi]\xi, \quad (25)$$

where  $\mathcal{R}[\Phi]$  is supersymmetric variation of  $\Phi$  and  $\xi$  is infinitesimal parameter of transformation. This observation gives us a freedom to generalize the supersymmetry transformation in the same fashion as discussed above by making the parameter,  $\xi$ , finite and field-dependent. We first define the infinitesimal field-dependent transformation as

$$\frac{d\Phi(\sigma, \kappa)}{d\kappa} = \mathcal{R}[\Phi(\sigma, \kappa)] \Theta'[\Phi(\sigma, \kappa)], \quad (26)$$

where the  $\Theta'[\Phi(\sigma, \kappa)]$  is an infinitesimal field-dependent parameter and  $\sigma$  is a parameter which parametrizes the base space of sigma models. The generalized supersymmetry ( $\delta_g$ ) with the finite field-dependent parameter then can be obtained by integrating the above transformation from  $\kappa = 0$  to  $\kappa = 1$ , as follows:

$$\delta_g \Phi(\sigma) \equiv \Phi(\sigma, \kappa = 1) - \Phi(\sigma, \kappa = 0) \\ = \mathcal{R}[\Phi(\sigma)] \Theta[\Phi(\sigma)], \quad (27)$$

where  $\Theta[\Phi(\sigma)]$  is the finite field-dependent parameter constructed from its infinitesimal version using (6) written in base space. Under such generalized supersymmetry transformation with finite field-dependent parameter the measure of partition function will not be invariant and will contribute some non-trivial terms to the partition function in general.

The Jacobian of the path integral measure ( $\mathcal{D}\Phi$ ) in the functional integral for such transformations is then evaluated for some particular choices of the finite field-dependent parameter,  $\Theta[\Phi(\sigma)]$ , as

$$\mathcal{D}\Phi' = J(\kappa) \mathcal{D}\Phi(\kappa). \quad (28)$$

Now we replace the Jacobian  $J(\kappa)$  of the path integral measure as

$$J(\kappa) \mapsto e^{-S[\Phi(\sigma, \kappa)]}, \quad (29)$$

by paying the cost that the given condition (10) must be satisfied where  $S[\Phi]$  is some local functional of fields satisfying initial boundary condition given in (11).

Moreover, the infinitesimal change in Jacobian,  $J(\kappa)$ , as before,

$$\frac{d}{d\kappa} \ln J(\kappa) = - \int d^m \sigma \left[ \pm \sum_i \mathcal{R}[\Phi^i(\sigma)] \frac{\partial \Theta'[\Phi(\sigma, \kappa)]}{\partial \Phi^i(\sigma, \kappa)} \right], \quad (30)$$

where, for bosonic fields, + sign is used and for fermionic fields, – sign is used.

### 3. Sigma models

In this section, we will use the supersymmetric FFBRST mechanism to generate the actions for two distinct sigma models. First, we discuss the sigma model on a curved target space and then a topological sigma model on quaternionic manifolds.

#### 3.1. Sigma model on a curved target space

To discuss the sigma model, let us start by considering the real bosonic field  $\phi^i(\sigma)$  corresponding to coordinates on a Riemannian target manifold with metric  $g_{ij}$  where the coordinate  $\sigma$  parametrizes the one-dimensional base space. This theory is supersymmetrized by considering two more real fermionic fields  $\psi_i(\sigma)$  and  $\eta_i(\sigma)$  and one Lagrange multiplier (bosonic) field  $B_i(\sigma)$ . Now, the infinitesimal supersymmetry transformations parametrized by a global Grassmann parameter  $\xi$  are given by [6]

$$\delta \phi^i = -\psi^i \xi, \\ \delta \psi^i = 0, \\ \delta \eta_i = (B_i - \eta_j \Gamma^j_{ik} \psi^k) \xi, \\ \delta B_i = - \left( B_j \Gamma^j_{ik} \psi^k - \frac{1}{2} \eta_j R^j_{ilk} \psi^l \psi^k \right) \xi, \\ \delta \Gamma^j_{ik} = \partial_m \Gamma^j_{ik} \psi^m \xi, \\ \delta R^j_{ilk} = \partial_m R^j_{ilk} \psi^m \xi, \quad (31)$$

where, in terms of affine connection  $\Gamma^j_{ik}$ , the Riemannian curvature tensor  $R^i_{jkl}$  is defined by:

$$R^i_{jkl} = \partial_k \Gamma^i_{jl} - \partial_l \Gamma^i_{jk} + \Gamma^i_{mk} \Gamma^m_{jl} - \Gamma^i_{ml} \Gamma^m_{jk}. \quad (32)$$

For any general fields  $f(\sigma)$  and  $g(\sigma)$ , the supersymmetric operator  $\delta$  acts on the composite field  $f \cdot g$  as follows  $(\delta f) \cdot g + f \cdot (\delta g)$ . With this definition, the nilpotency of operator  $\delta$  (i.e.,  $\delta^2 = 0$ ) can be proved easily in the following manner:

$$\delta^2 \phi^i = \delta \psi^i = 0, \\ \delta^2 \eta_i = \delta B_i - \delta \eta_j \Gamma^j_{ik} \psi^k - \eta_j \delta \Gamma^j_{ik} \psi^k = 0, \\ \delta^2 B_i = -\delta B_j \Gamma^j_{ik} \psi^k - B_j \delta \Gamma^j_{ik} \psi^k + \frac{1}{2} \delta \eta_j R^j_{ilk} \psi^l \psi^k \\ + \frac{1}{2} \eta_j \delta R^j_{ilk} \psi^l \psi^k = 0, \\ \delta^2 \Gamma^j_{ik} = \partial_m \partial_n \Gamma^j_{ik} \psi^n \psi^m = 0, \\ \delta R^j_{ilk} = \partial_m \partial_n R^j_{ilk} \psi^n \psi^m = 0. \quad (33)$$

Now, the supersymmetric action for the sigma model in one dimension, which remains invariant under the above fermion transformations, is given by [6]

$$S = \alpha \int d\sigma \left[ B_i N^i(\phi) - \frac{1}{2} g^{ij} B_i B_j - \eta_i \nabla_k N^i \psi^k \right. \\ \left. + \frac{1}{4} R_{jlmk} \eta^j \eta^l \psi^m \psi^k \right], \quad (34)$$

where  $N^i(\phi)$  denotes an arbitrary gauge-fixing condition for the bosonic field  $\phi^i$  and  $\alpha$  is a coupling constant. Here we note that the supersymmetric invariant observables do not depend on the choice of  $\alpha$ . The symbol  $\nabla_k$  indicates the general target space covariant derivative. For the sigma model the most convenient gauge-fixing condition is [6],

$$N^i(\phi) = \frac{d\phi^i}{d\sigma}. \quad (35)$$

For this particular choice, the above action reduces to the form:

$$S = \alpha \int d\sigma \left[ B_i \frac{d\phi^i}{d\sigma} - \frac{1}{2} g^{ij} B_i B_j - \eta_i \left( \frac{d\psi^i}{d\sigma} + \Gamma^i_{kj} \frac{d\phi^k}{d\sigma} \psi^j \right) \right. \\ \left. + \frac{1}{4} R_{jlmk} \eta^j \eta^l \psi^m \psi^k \right]. \quad (36)$$

The generalized supersymmetric BRST transformation for one-dimensional sigma model on a curved target space is constructed by

$$\delta_g \phi^i = -\psi^i \Theta[\Phi], \\ \delta_g \psi^i = 0, \\ \delta_g \eta_i = (B_i - \eta_j \Gamma^j_{ik} \psi^k) \Theta[\Phi], \\ \delta_g B_i = - \left( B_j \Gamma^j_{ik} \psi^k - \frac{1}{2} \eta_j R^j_{ilk} \psi^l \psi^k \right) \Theta[\Phi], \\ \delta_g \Gamma^j_{ik} = \partial_m \Gamma^j_{ik} \psi^m \Theta[\Phi], \\ \delta_g R^j_{ilk} = \partial_m R^j_{ilk} \psi^m \Theta[\Phi], \quad (37)$$

where  $\Theta[\Phi]$  is the general finite field-dependent parameter. For instance, we choose a specific  $\Theta[\Phi]$  obtained from the following infinitesimal field-dependent parameter using relation (6):

$$\Theta'[\eta, \phi, B] = -\alpha \int d\sigma \eta_i \left( \frac{d\phi^i}{d\sigma} - \frac{1}{2} g^{ij} B_j \right). \quad (38)$$

The infinitesimal change of Jacobian of the path integral measure is calculated by exploiting relation (30) as

$$\begin{aligned} & \frac{d}{d\kappa} \ln J(\kappa) \\ &= \alpha \int d\sigma \left[ -B_i \frac{d\phi^i}{d\sigma} + \frac{1}{2} g^{ij} B_i B_j + \eta_i \left( \frac{d\psi^i}{d\sigma} + \Gamma^i_{kj} \frac{d\phi^k}{d\sigma} \psi^j \right) \right. \\ & \quad \left. - \frac{1}{4} R_{jlmk} \eta^j \eta^l \psi^m \psi^k \right]. \end{aligned} \quad (39)$$

Now, we make an ansatz for the arbitrary functional  $S$  which appears in the expression (exponent) of the Jacobian (29) as

$$\begin{aligned} S[\phi(\sigma, \kappa), \kappa] \\ &= \int d\sigma \left[ \zeta_1(\kappa) B_i \frac{d\phi^i}{d\sigma} + \zeta_2(\kappa) g^{ij} B_i B_j \right. \\ & \quad + \zeta_3(\kappa) \eta_i \left( \frac{d\psi^i}{d\sigma} + \Gamma^i_{kj} \frac{d\phi^k}{d\sigma} \psi^j \right) \\ & \quad \left. + \zeta_4(\kappa) R_{jlmk} \eta^j \eta^l \psi^m \psi^k \right], \end{aligned} \quad (40)$$

where  $\zeta_1, \zeta_2, \zeta_3$  and  $\zeta_4$  are  $\kappa$ -dependent constants which vanish at  $\kappa = 0$ . The existence of the above functional is valid when it satisfies the essential requirement given in (10) along with (39). This leads to the following condition:

$$\begin{aligned} & \int d\sigma \left[ \left( \frac{d\zeta_1}{d\kappa} - \alpha \right) B_i \frac{d\phi^i}{d\sigma} + \left( \frac{d\zeta_2}{d\kappa} + \frac{1}{2} \alpha \right) g^{ij} B_i B_j \right. \\ & \quad + \left( \frac{d\zeta_3}{d\kappa} + \alpha \right) \eta_i \left( \frac{d\psi^i}{d\sigma} + \Gamma^i_{kj} \frac{d\phi^k}{d\sigma} \psi^j \right) \\ & \quad + \left( \frac{d\zeta_4}{d\kappa} - \frac{1}{4} \alpha \right) R_{jlmk} \eta^j \eta^l \psi^m \psi^k \\ & \quad + (\zeta_2 + 2\zeta_4) \eta_j R^j_{ilk} \psi^l \psi^k B^i \Theta'[\phi] \\ & \quad - (\zeta_1 + \zeta_3) \left( B_i \frac{d\psi^i}{d\sigma} + B_j \Gamma^j_{ik} \psi^k \frac{d\phi^i}{d\sigma} \right. \\ & \quad \left. - \frac{1}{2} \eta_j R^j_{ilk} \psi^l \psi^k \frac{d\phi^i}{d\sigma} \right) \Theta'[\phi] \Big] = 0, \end{aligned} \quad (41)$$

where we have used the antisymmetry of the Grassmann variables and Bianchi identity of Riemann tensor. The comparison of various terms on both sides yields the following constraints on the parameters  $\zeta_i(\kappa)$ , where  $i = 1, 2, 3, 4$ :

$$\frac{d\zeta_1(\kappa)}{d\kappa} - \alpha = 0, \quad (42)$$

$$\frac{d\zeta_2(\kappa)}{d\kappa} + \frac{1}{2} \alpha = 0, \quad (43)$$

$$\frac{d\zeta_3(\kappa)}{d\kappa} + \alpha = 0, \quad (44)$$

$$\frac{d\zeta_4(\kappa)}{d\kappa} - \frac{1}{4} \alpha = 0, \quad (45)$$

$$\zeta_1(\kappa) + \zeta_3(\kappa) = 0, \quad (46)$$

$$\zeta_2(\kappa) + 2\zeta_4(\kappa) = 0. \quad (47)$$

The solutions of the above differential equations given in (42)–(45) are

$$\begin{aligned} \zeta_1(\kappa) &= \alpha\kappa, & \zeta_2 &= -\frac{1}{2}\alpha\kappa, & \zeta_3(\kappa) &= -\alpha\kappa, \\ \zeta_4(\kappa) &= \frac{1}{4}\alpha\kappa. \end{aligned} \quad (48)$$

These solutions are also consistent with relations (46) and (47). Therefore, with these identifications of  $\zeta_i$ , action  $S$  simplifies as

$$\begin{aligned} S[\phi(\sigma, \kappa), \kappa] \\ &= \alpha\kappa \int d\sigma \left[ B_i \frac{d\phi^i}{d\sigma} - \frac{1}{2} g^{ij} B_i B_j - \eta_i \left( \frac{d\psi^i}{d\sigma} + \Gamma^i_{kj} \frac{d\phi^k}{d\sigma} \psi^j \right) \right. \\ & \quad \left. + \frac{1}{4} R_{jlmk} \eta^j \eta^l \psi^m \psi^k \right], \end{aligned} \quad (49)$$

which vanishes at  $\kappa = 0$ . However, at  $\kappa = 1$  (under generalized supersymmetry transformation), it takes the following form

$$\begin{aligned} S[\phi(\sigma, 1), 1] \\ &= \alpha \int d\sigma \left[ B_i \frac{d\phi^i}{d\sigma} - \frac{1}{2} g^{ij} B_i B_j - \eta_i \left( \frac{d\psi^i}{d\sigma} + \Gamma^i_{kj} \frac{d\phi^k}{d\sigma} \psi^j \right) \right. \\ & \quad \left. + \frac{1}{4} R_{jlmk} \eta^j \eta^l \psi^m \psi^k \right], \end{aligned} \quad (50)$$

which exactly coincides with the effective action (36) for the sigma model on curved target space in one dimension. This shows that the effective action for the sigma model on curved target space emerges naturally through the Jacobian of the path integral measure under generalized supersymmetric transformation. Now if we apply again the FFBRST transformation with appropriate choice of finite field-dependent parameter, we can get the sigma model in different gauges.

### 3.2. Topological sigma model

In this subsection we discuss the topological sigma model for hyper-Kähler map. For this purpose we start by defining a map  $\phi: \mathcal{M} \rightarrow \mathcal{N}$  from a Riemannian world-manifold  $\mathcal{M}$  to a Riemannian target-manifold  $\mathcal{N}$  which deals with the homotopy classes of the map. This map is described by an action

$$S = \int_{\mathcal{M}} d^m \sigma \sqrt{g(\sigma)} g^{\alpha\beta}(\sigma) \partial_\alpha \phi^i \partial_\beta \phi^j h_{ij}(\phi), \quad (51)$$

where  $m = \dim \mathcal{M}$ ,  $g^{\alpha\beta}(\sigma)$  is the metric of the world-manifold  $\mathcal{M}$  and  $h_{ij}(\phi)$  is the metric of target-manifold  $\mathcal{N}$ . Here Greek indices  $\alpha, \beta = 1, 2, \dots, m$  denote the world indices and indices  $i, j = 1, 2, \dots, 4n$  refer to the target ones where  $\dim \mathcal{N} = 4n$  is fixed. This action is topologically invariant under any continuous deformation,  $\phi \rightarrow \phi + \delta\phi$ , due to the large symmetry required by it. Therefore topological sigma model is intrinsically a quantum field theory. This large symmetry is BRST-quantized [15,18] in the usual ways and the gauge is fixed by choosing suitable representatives in the homotopy classes of the maps  $\phi$ .

The supersymmetric BRST-quantization of the theory is achieved as follows. First of all we introduce topological ghosts  $\psi^i$  as well as topological antighosts  $\eta^i_\alpha$  and Lagrange multipliers  $B^i_\alpha$  corresponding to the gauge-fixing in the theory. Here an extra index  $\alpha$  corresponds to the directions in the base space. These antighosts and Lagrange multipliers are required to satisfy the following duality condition

$$\eta^i_\alpha - \frac{1}{3} (j_u)_\alpha{}^\beta \eta^j_\beta (j_u)_j{}^i = 0, \quad B^i_\alpha + \frac{1}{3} (j_u)_\alpha{}^\beta B^j_\beta (j_u)_j{}^i = 0, \quad (52)$$

where  $j_u (j_u)$  are called the almost quaternionic (1,1)-tensors of  $\mathcal{M} (\mathcal{N})$  with  $u = 1, 2, 3$ . Now, the nilpotent supersymmetry transformations are constructed as [17]

$$\begin{aligned} \delta\phi^i &= -\psi^i \xi, \\ \delta\psi^i &= 0, \end{aligned}$$



$$\begin{aligned}
\delta\eta_\alpha^i &= B_\alpha^i \xi - \Gamma^i_{jk} \psi^j \eta_\alpha^k \xi - \frac{1}{4} (j_u)_\alpha^\beta D_k(J_u)_j^i \psi^k \eta_\beta^j \xi, \\
\delta B_\alpha^i &= -\frac{1}{2} R_{jk}^i \psi^j \psi^k \eta_\alpha^l \xi + \Gamma^i_{jk} \psi^j B_\alpha^k \xi \\
&\quad + \frac{1}{4} (j_u)_\alpha^\beta D_k(J_u)_j^i \psi^k B_\beta^j \xi \\
&\quad - \frac{1}{4} (j_u)_\alpha^\beta D_m D_k(J_u)_j^i \psi^m \psi^k \eta_\beta^j \xi \\
&\quad + \frac{1}{16} D_k(J_u)_j^i D_l(J_u)_m^j \psi^k \psi^l \eta_\alpha^m \xi \\
&\quad - \frac{1}{16} \epsilon_{uvz} (j_z)_\alpha^\beta D_k(J_u)_j^i D_l(J_v)_m^j \psi^k \psi^l \eta_\beta^m \xi, \quad (53)
\end{aligned}$$

where  $\xi$  is global anticommuting parameter. Here the covariant derivative of  $\psi^i$  is defined by

$$D_\alpha \psi^i = \partial_\alpha \psi^i + \Gamma^i_{jk} \partial_\alpha \phi^j \psi^k. \quad (54)$$

Now, with these introductions the supersymmetric action for topological sigma model is constructed by [17]

$$S = S_{\text{bose}} + S_{\text{fermi}}, \quad (55)$$

where

$$\begin{aligned}
S_{\text{bose}} &= \int_{\mathcal{M}} d^m \sigma \sqrt{g} g^{\alpha\beta} h_{ij} B_\alpha^i \left( \partial_\beta \phi^j - \frac{1}{8} B_\beta^j \right), \\
S_{\text{fermi}} &= \int_{\mathcal{M}} d^m \sigma \sqrt{g} \left[ -g^{\alpha\beta} h_{ij} \eta_\alpha^i D_\beta \psi^j + \frac{1}{16} R_{ijkl} g^{\alpha\beta} \eta_\alpha^i \eta_\beta^j \psi^k \psi^l \right. \\
&\quad + \frac{1}{4} \eta_\alpha^m (j_u)^{\beta\alpha} D_k(J_u)_{mj} \partial_\beta \phi^j \psi^k \\
&\quad + \frac{1}{32} \eta_\alpha^i \eta_\beta^l (j_u)^{\alpha\beta} D_m D_k(J_u)_{li} \psi^m \psi^k \\
&\quad - \frac{1}{128} g^{\alpha\beta} \eta_\alpha^i \eta_\beta^m D_k(J_u)_{li} D_n(J_u)_m^l \psi^k \psi^n \\
&\quad \left. + \frac{1}{128} \eta_\alpha^i \eta_\beta^m \epsilon_{uvz} (j_z)^{\alpha\beta} D_k(J_u)_{li} D_n(J_v)_m^l \psi^k \psi^n \right], \quad (56)
\end{aligned}$$

which remains invariant under the supersymmetry transformations given in (53).

The supersymmetry of topological sigma model given in (53) is generalized as

$$\begin{aligned}
\delta\phi^i &= -\psi^i \Theta[\phi], \\
\delta\psi^i &= 0, \\
\delta\eta_\alpha^i &= B_\alpha^i \Theta[\phi] - \Gamma^i_{jk} \psi^j \eta_\alpha^k \Theta[\phi] \\
&\quad - \frac{1}{4} (j_u)_\alpha^\beta D_k(J_u)_j^i \psi^k \eta_\beta^j \Theta[\phi], \\
\delta B_\alpha^i &= -\frac{1}{2} R_{jk}^i \psi^j \psi^k \eta_\alpha^l \Theta[\phi] + \Gamma^i_{jk} \psi^j B_\alpha^k \Theta[\phi] \\
&\quad + \frac{1}{4} (j_u)_\alpha^\beta D_k(J_u)_j^i \psi^k B_\beta^j \Theta[\phi] \\
&\quad - \frac{1}{4} (j_u)_\alpha^\beta D_m D_k(J_u)_j^i \psi^m \psi^k \eta_\beta^j \Theta[\phi] \\
&\quad + \frac{1}{16} D_k(J_u)_j^i D_l(J_u)_m^j \psi^k \psi^l \eta_\alpha^m \Theta[\phi] \\
&\quad - \frac{1}{16} \epsilon_{uvz} (j_z)_\alpha^\beta D_k(J_u)_j^i D_l(J_v)_m^j \psi^k \psi^l \eta_\beta^m \Theta[\phi], \quad (57)
\end{aligned}$$

where  $\Theta[\Phi]$  is an arbitrary finite field-dependent parameter. However, it can be specified to have some particular values. For example, we choose the  $\Theta[\phi]$  obtained from the following infinitesimal field-dependent parameter using relation (6):

$$\Theta'[\eta, \phi, B] = - \int_{\mathcal{M}} d^m \sigma \sqrt{g} g^{\alpha\beta} h_{ij} \eta_\alpha^i \left( \partial_\beta \phi^j - \frac{1}{8} B_\beta^j \right). \quad (58)$$

Now, exploiting relation (30), the infinitesimal change of Jacobian of the path integral measure is calculated as

$$\begin{aligned}
&\frac{d}{d\kappa} \ln J(\kappa) \\
&= \int_{\mathcal{M}} d^m \sigma \sqrt{g} \left[ -g^{\alpha\beta} h_{ij} B_\alpha^i \left( \partial_\beta \phi^j - \frac{1}{8} B_\beta^j \right) + g^{\alpha\beta} h_{ij} \eta_\alpha^i D_\beta \psi^j \right. \\
&\quad - \frac{1}{16} R_{ijkl} g^{\alpha\beta} \eta_\alpha^i \eta_\beta^j \psi^k \psi^l - \frac{1}{4} \eta_\alpha^m (j_u)^{\beta\alpha} D_k(J_u)_{mj} \partial_\beta \phi^j \psi^k \\
&\quad - \frac{1}{32} \eta_\alpha^i \eta_\beta^l (j_u)^{\alpha\beta} D_m D_k(J_u)_{li} \psi^m \psi^k \\
&\quad + \frac{1}{128} g^{\alpha\beta} \eta_\alpha^i \eta_\beta^m D_k(J_u)_{li} D_n(J_u)_m^l \psi^k \psi^n \\
&\quad \left. - \frac{1}{128} \eta_\alpha^i \eta_\beta^m \epsilon_{uvz} (j_z)^{\alpha\beta} D_k(J_u)_{li} D_n(J_v)_m^l \psi^k \psi^n \right]. \quad (59)
\end{aligned}$$

Further, we make an arbitrary ansatz for the functional  $S[\Phi]$  (29) having similar terms as in RHS of (59). Henceforth,  $S[\Phi]$  is defined by

$$\begin{aligned}
S[\Phi(\sigma, \kappa, \kappa)] &= \int_{\mathcal{M}} d^m \sigma \left[ \zeta_1(\kappa) g^{\alpha\beta} h_{ij} B_\alpha^i \partial_\beta \phi^j + \zeta_2(\kappa) g^{\alpha\beta} h_{ij} B_\alpha^i B_\beta^j \right. \\
&\quad + \zeta_3(\kappa) g^{\alpha\beta} h_{ij} \eta_\alpha^i D_\beta \psi^j + \zeta_4(\kappa) R_{ijkl} g^{\alpha\beta} \eta_\alpha^i \eta_\beta^j \psi^k \psi^l \\
&\quad + \zeta_5(\kappa) \eta_\alpha^m (j_u)^{\beta\alpha} D_k(J_u)_{mj} \partial_\beta \phi^j \psi^k \\
&\quad + \zeta_6(\kappa) \eta_\alpha^i \eta_\beta^l (j_u)^{\alpha\beta} D_m D_k(J_u)_{li} \psi^m \psi^k \\
&\quad + \zeta_7(\kappa) g^{\alpha\beta} \eta_\alpha^i \eta_\beta^m D_k(J_u)_{li} D_n(J_u)_m^l \psi^k \psi^n \\
&\quad \left. + \zeta_8(\kappa) \eta_\alpha^i \eta_\beta^m \epsilon_{uvz} (j_z)^{\alpha\beta} D_k(J_u)_{li} D_n(J_v)_m^l \psi^k \psi^n \right], \quad (60)
\end{aligned}$$

where  $\zeta_i(\kappa)$ ,  $i = 1, 2, \dots, 8$ , are  $\kappa$ -dependent constants satisfying initial boundary conditions. Eqs. (59) and (60) together with condition (10) yield the following differential equations

$$\frac{d\zeta_1(\kappa)}{d\kappa} - \sqrt{g} = 0, \quad (61)$$

$$\frac{d\zeta_2(\kappa)}{d\kappa} + \frac{1}{8} \sqrt{g} = 0, \quad (62)$$

$$\frac{d\zeta_3(\kappa)}{d\kappa} + \sqrt{g} = 0, \quad (63)$$

$$\frac{d\zeta_4(\kappa)}{d\kappa} - \frac{1}{16} \sqrt{g} = 0, \quad (64)$$

$$\frac{d\zeta_5(\kappa)}{d\kappa} - \frac{1}{4} \sqrt{g} = 0, \quad (65)$$

$$\frac{d\zeta_6(\kappa)}{d\kappa} - \frac{1}{32} \sqrt{g} = 0, \quad (66)$$

$$\frac{d\zeta_7(\kappa)}{d\kappa} + \frac{1}{128} \sqrt{g} = 0, \quad (67)$$

$$\frac{d\zeta_8(\kappa)}{d\kappa} - \frac{1}{128} \sqrt{g} = 0. \quad (68)$$

The above linear differential equations are exactly solvable. Their solutions satisfying the initial conditions  $\xi_i(\kappa = 0) = 0$ ,  $i = 1, 2, 3, 4$ , are

$$\begin{aligned}\zeta_1(\kappa) &= \sqrt{g}\kappa, & \zeta_2(\kappa) &= -\frac{1}{8}\sqrt{g}\kappa, & \zeta_3(\kappa) &= -\sqrt{g}\kappa, \\ \zeta_4(\kappa) &= \frac{1}{16}\sqrt{g}\kappa, & \zeta_5(\kappa) &= \frac{1}{4}\sqrt{g}\kappa, & \zeta_6(\kappa) &= \frac{1}{32}\sqrt{g}\kappa, \\ \zeta_7(\kappa) &= -\frac{1}{128}\sqrt{g}\kappa, & \zeta_8(\kappa) &= \frac{1}{128}\sqrt{g}\kappa.\end{aligned}\quad (69)$$

With these values of constants, the functional  $S[\phi(\sigma, \kappa)]$  reduces to

$$\begin{aligned}S[\Phi(\sigma, \kappa), \kappa] &= \kappa \int_{\mathcal{M}} d^m \sigma \sqrt{g} \left[ g^{\alpha\beta} h_{ij} B_\alpha^i \left( \partial_\beta \phi^j - \frac{1}{8} B_\beta^j \right) - g^{\alpha\beta} h_{ij} \eta_\alpha^i D_\beta \psi^j \right. \\ &\quad + \frac{1}{16} R_{ijkl} g^{\alpha\beta} \eta_\alpha^i \eta_\beta^j \psi^k \psi^l + \frac{1}{4} \eta_\alpha^m (j_u)^{\beta\alpha} D_k (J_u)_{mj} \partial_\beta \phi^j \psi^k \\ &\quad + \frac{1}{32} \eta_\alpha^i \eta_\beta^l (j_u)^{\alpha\beta} D_m D_k (J_u)_{li} \psi^m \psi^k \\ &\quad - \frac{1}{128} g^{\alpha\beta} \eta_\alpha^i \eta_\beta^m D_k (J_u)_{li} D_n (J_u)_{m}^l \psi^k \psi^n \\ &\quad \left. + \frac{1}{128} \eta_\alpha^i \eta_\beta^m \epsilon_{uvz} (j_z)^{\alpha\beta} D_k (J_u)_{li} D_n (J_v)_{m}^l \psi^k \psi^n \right],\end{aligned}\quad (70)$$

which vanishes at  $\kappa = 0$ . However, for  $\kappa = 1$ , it becomes

$$\begin{aligned}S[\Phi(\sigma, 1), 1] &= \int_{\mathcal{M}} d^m \sigma \sqrt{g} \left[ g^{\alpha\beta} h_{ij} B_\alpha^i \left( \partial_\beta \phi^j - \frac{1}{8} B_\beta^j \right) - g^{\alpha\beta} h_{ij} \eta_\alpha^i D_\beta \psi^j \right. \\ &\quad + \frac{1}{16} R_{ijkl} g^{\alpha\beta} \eta_\alpha^i \eta_\beta^j \psi^k \psi^l + \frac{1}{4} \eta_\alpha^m (j_u)^{\beta\alpha} D_k (J_u)_{mj} \partial_\beta \phi^j \psi^k \\ &\quad + \frac{1}{32} \eta_\alpha^i \eta_\beta^l (j_u)^{\alpha\beta} D_m D_k (J_u)_{li} \psi^m \psi^k \\ &\quad - \frac{1}{128} g^{\alpha\beta} \eta_\alpha^i \eta_\beta^m D_k (J_u)_{li} D_n (J_u)_{m}^l \psi^k \psi^n \\ &\quad \left. + \frac{1}{128} \eta_\alpha^i \eta_\beta^m \epsilon_{uvz} (j_z)^{\alpha\beta} D_k (J_u)_{li} D_n (J_v)_{m}^l \psi^k \psi^n \right],\end{aligned}\quad (71)$$

which is the exact expression of the supersymmetric topological sigma model (55) in  $m$  dimensions. Therefore, we generated the effective action for supersymmetric topological sigma model by calculating the Jacobian of the path integral under generalized supersymmetry transformations with appropriate transformation parameter. Further, we observe that under further generalized supersymmetry with appropriate field-dependent parameter we can map the topological sigma model from one gauge to another.

#### 4. Conclusions

In this paper, we have described the mechanism of generalized BRST transformation to establish the connection between two different gauges of Maxwell theory. In the same fashion, we have proposed the idea behind generalizing supersymmetry. We have generalized the BRST supersymmetry by allowing the transformation parameter to be finite and field-dependent. The generalized supersymmetry retains the invariance at the level of the action only, however, the generating functional does not. The obvious reason for this is that the path integral measure is not invariant

under the transformation. We have shown that under such generalized supersymmetry, the path integral measure of functional integral changes non-trivially. We have sketched a novel feature originating from such non-trivial Jacobian under generalized supersymmetry. With suitable choices of finite and field-dependent transformation parameters, the Jacobian generates the supersymmetric actions corresponding to sigma models. In fact the Jacobian reproduces the well known supersymmetric actions of sigma models.

It is useful to note that not all supersymmetric actions may be generated in this manner. As discussed earlier, only those actions that are BRST exact may be obtained. This is essentially tied to the fact that the Jacobian of FFBRS transformation is BRST exact.

The present analysis highlights the important role of symmetry in the abstraction of supersymmetric actions. As is well known, for non-supersymmetric theories, gauge invariance is crucial for the obtention of actions. The calculation of the one loop effective action using a gauge invariant regularization provides a striking example in this context. In  $(1+1)$  dimensions the computation can be done exactly and yields the Schwinger model. In  $(2+1)$  (or higher) dimensions the result cannot be obtained exactly and one takes recourse to the derivative expansion. The first term is the (single derivative) Chern–Simons term, the second is the (two derivative) Maxwell term and so on. For supersymmetric theories, gauge invariance gets replaced by BRST invariance. Our analysis shows in a precise way the role of this invariance in obtaining a certain class of supersymmetric actions. This was explicitly shown for two supersymmetric sigma models – a sigma model in one dimension and a topological sigma model in general dimensions. It illustrates the robustness of the technique in the sense that it may be applied in quite distinct situations. We note that under the action of further generalized supersymmetry transformations with appropriate transformation parameters we will be able to connect the supersymmetric sigma models in different gauges, exactly as was discussed for the Maxwell theory. We hope this formulation will help to systematically construct the supersymmetric actions for sigma models in an elegant manner as well as provides a deeper understanding.

Let us discuss the arbitrariness in this scheme. It is contained in the choice of the FFBRS parameter  $\Theta$  defined generally in (6) and in (38) and (58) for the specific models under consideration. Once this choice is made the rest follows systematically. The point is that once  $\Theta$  is defined, the infinitesimal change of the Jacobian is calculated from the specified formula (29). The structure of this change determines the ansatz to be adopted for the functional that appears in the exponent of the Jacobian (28). This eventually yields the final answer. The choice of the  $\Theta$  parameter is somewhat akin to choosing a good gauge. A judicious choice of this parameter is important to get meaningful results.

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